Asymptotic Performance of Digital Computation

Alexander Zeltsman*
(Dated: April 10, 2008)

Quantum measurement nature of the digital computation, implies a tradeoff between speed, power consumption, and processing parallelism. Asymptotic expression of their relationships, which has been obtained in framework of a simple model, helps to shed some light on the future trends of performance of digital computers, as they are poised to probe area ruled by the quantum mechanical uncertainty.

INTRODUCTION

Question of the physical limitations on digital computation speed has attracted considerable interest, as we want predict the future of exponentially rapid progress of computer hardware, reflected in Moore's Law.

Quantum Mechanical limitations arise from the fact that digital processing is substantially classical process. The digital processing relies on the information bits being in definite state at definite time - which requires equivalent of some sort of synchronized measurement. Being currently overshadowed by the technological issues, the quantum limitations will quickly rise in importance as we approach their bounds.

MODEL

The model uses original Quantum Uncertainty Principle formulation [1], to describe single bit setting:

$$\Delta E \cdot \Delta t \simeq h \tag{1}$$

We prefer its consistency to the more commonly used [2][3]

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} = \frac{h}{4\pi} \tag{2}$$

which would produce similar asymptotic estimates , but reflects slightly different meaning of ΔE and Δt, gives reduced lower limit for $\Delta E \cdot \Delta t$, and does not suggest any upper limit.

dE is interpreted as average amount of energy that must be dissipated at each setting of a bit. It is an expense of digitizing, paid for obtaining time synchronization better than dt in the quantum microscopic world.

Model parameters and values

Δt - time of bit setting, minimum time of operation

ΔE - energy, dissipated at bit setting/read

w - operational bits settings per floating point operation

S - full number of bit settings/reads per one operational bit setting, an overhead factor

N - number of processors in the system

wS - total number of bit settings/reads, involved in an operation on single processor

wSN - total number of bit operations, performed by the system each Δt

E_{total} - total energy, dissipated by the system every Δt

P - total power, consumed by the system

f - total operations per second performed

T - absolute temperature of processor

Model Computations

In the model

$$f = \frac{N}{\Delta t} \tag{3}$$

Using (1)

$$\Delta E = \frac{h}{\Delta t} \tag{4}$$

which yields:

$$E_{total} = \frac{wSNh}{\Delta t} \tag{5}$$

$$P = \frac{E_{total}}{\Delta t} = \frac{wSNh}{\Delta t^2} \tag{6}$$

finally, substituting (3) into (6), we obtain:

$$P = \frac{wShf^2}{N} \tag{7}$$

$$f = \sqrt{\frac{P \cdot N}{wSh}} \tag{8}$$

The obtained expressions (7) and (8) are suitable for asymptotic estimates. Although derived in the framework of simple model, those are universal relationships, and they can serve, as a base for prediction of asymptotic computer performance. Some parameters can be expected not to change much, and we estimate them using their current values. Mistakes of such estimates have only superficial influence on the asymptotic projections.

The basic parameter estimates:

$$w =_{est} 100, S =_{est} 10, T =_{est} 300$$

$$P_{singleprocessor} =_{est} 100, P_{multipleprocessor} =_{est} 10^8$$

Single Processor Systems

Quantum limitations:

$$\Delta E \simeq \frac{h}{\Delta t} \quad (9)$$

$$P = wS \cdot \frac{1}{\Delta t} \cdot \Delta E \simeq wShf_q{}^2 \quad (10)$$

$$f_q \simeq \sqrt{\frac{P}{wSh}} = \sqrt{\frac{P}{wS}} \cdot 3.9 \cdot 10^{16} =_{est} 1.23 \cdot 10^{16} \quad (11)$$

Termal limitations:

$$\Delta E \simeq kT \quad (12)$$

$$P = wS \cdot \frac{1}{\Delta t} \cdot \Delta E \simeq wSkTf_T \quad (13)$$

$$f_T \simeq \frac{P}{wSkT} = \frac{P}{wS} \cdot \frac{1}{T} \cdot 7.25 \cdot 10^{22} =_{est} 2.42 \cdot 10^{19} \quad (14)$$

We must conclude that heat never overtakes quantum expenses of synchronization as the main asymptotic disruption to computation.

Using single processor yearly performance growth 1.4 , performance at 2002 equal $3.16 GFLOP$ [4], we get 2050 as the estimate year, when upper limit of single processor peformance of $12 PFLOP$ will be achieved by the fastest processors.

Massively Parallel Systems

Performance of system can continue to grow due to increase in parallelism, even when quantum limitations play the defining role.
To find out when the existing pattern of growth is going to be interrupted due to quantum limitations, we use multiple processor yearly performance growth 1.82 , number of processors yearly growth 1.3 [4],

$N = 1.3 \cdot 10^5, P = 1.2 \cdot 10^6, f = 10^{15}$ in 2006 [4][5].

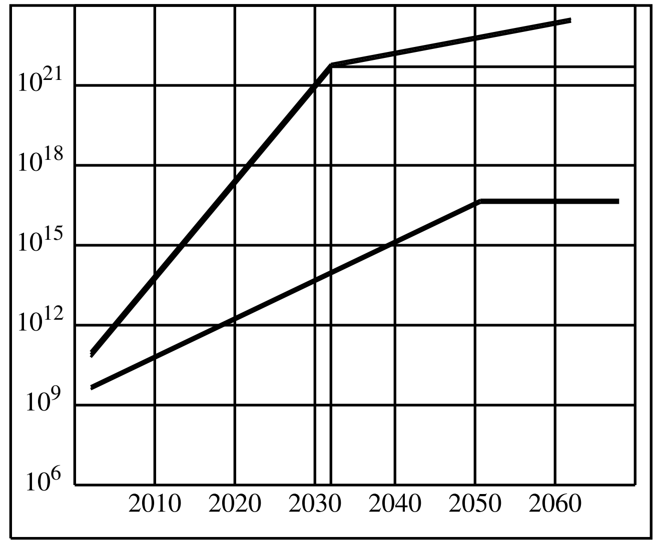

FIG. 1: Computation Performance Asymptotics.
The lower graph is for single processor systems

From this follows, that 2032 is the estimated year, when projected number of processors shall become insufficient for sustaining rate of performance growth.

At that critical point, $N = 1.2 \cdot 10^8, f = 5.78 \cdot 10^{21}$.

Single processor performance in the system is $f/N = 4.8 \cdot 10^{13}$, which is 256 times less than best single processors are expected to achieve.

From that point on, performance growth will be defined by increase in the number of processors, thus we will have asymptotically

$$f = 5.78 \cdot 10^{21} \cdot 1.14^{(year-2032)} \quad (15)$$

$$N = 1.2 \cdot 10^8 \cdot 1.3^{(year-2032)} \quad (16)$$

DISCUSSION

Although the obtained asymptotic estimates incorporate some assumptions that may be not applicable to particular system, like relatively stable system energy consumption over time, etc., they cteate a baseline for estimates of the asymptotic behavior of real systems. For example, some systems might be deemed to be so important, that extraordinary power expense may be acceptable, possibly temporary. So the asymptotic performance might be improved with plenty of power employed. Some systems can be intermediate in multiprocessing level, with various number of processors. Their asymptotic performance behavior will be intermediate between the cases that we have considered. Some systems may include two components, one with relatively small number of fast processors, and another employing massive parallelism - which would improve performance of such systems for tasks that require massive consecutive processing, compared to performance of standard massively parallel system. Yet another possible way to mitigate energy dissipation due to the quantum effects of synchronization, is to use analog components where appropriate.

* azeltsman2@comcast.net

[1] W. Heisenberg, Z. für Phys. **43, 172-198** (1927).
[2] S. Lloyd, Nature **406, 1047-1054** (2000).
[3] G. Cerofolini, Appl. Phys. **A 86, 23-29** (2007).
[4] H. D. Simon, NERSC Center, Analysis of TOP500 Data (2003).
[5] A. Gara, et al., IBM Journal of Research and Development, Blue Gene **49 2/3** (2005).